NASA TECHNICAL NOTE

NASA TN D-5112

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NASA TN D-5112

SYMBOL-ORIENTED FAILURE ANALYSIS IN MULTICOMPONENT SYSTEMS

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Cambridge, Mass.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • APRIL 1969

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ABSTRACT

Computer-oriented procedures are developed to assess the error in performance of electronic circuits and similar quasi-linear systems if one element in a multicomponent system fails. A tutorial introduction to tagging techniques is provided to make the presentation self-contained.

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SUMMARY

The classical work by Bode (ref. 1) on sensitivity analysis presents a wide range of results in symbolic form by setting one of the symbols equal to zero or infinity. The advent of computer-oriented symbol manipulation techniques required a more systematic approach to procedures anticipated by Bode's work.

During the past five years (1964-1969), techniques for sensitivity and variance analysis were developed for computer-oriented evaluation and design of lumped parameter circuits. These procedures were applied to optimization in circuit design, failure diagnostics, automatic control, and stochastic networks.

This investigation makes no claim to original contributions to theory, although some derivations are new, but has as its primary aim a balanced survey of existing results based on several hundred recent investigations:

1. Development of basic concepts, analytical approaches with emphasis on scope and limitation of procedures may be grouped according to major techniques:

    a. Sensitivity analysis — first-order and higher-order formulas (refs. 1, 2, and 3),
    b. Variance analysis — error and preassigned accuracy techniques (refs. 4, 5, and 6),
    c. Binary base tagging for lumped systems (refs. 7, 8, and 9).

2. Applications of tagging procedures to lumped parameter systems:

    a. Electrical circuits with active elements (refs. 10, 11, and 12),
    b. Stochastic networks applied to models of processes (refs. 13, 14, and 15),
    c. Tolerance analysis and Monte Carlo techniques (refs. 16, 17, and 18).

3. Surveys and summaries primarily with tutorial objectives:

   a. Textbooks, reference works and handbooks (refs. 19, 20, and 21),
   b. Topical reviews and expositions (refs. 22, 23, and 24),
   c. Tutorial surveys (refs. 25, 26, and 27).

It is desirable, therefore, to identify and summarize the common analytical foundations contained in the current technical literature by developing these concepts and procedures in an orderly form and as a coherent approach.

Each of the references cited will provide, in turn, substantial bibliographical information and, for this reason, is intended only as a sample of the extensive recent literature related to computer-oriented sensitivity and variance analysis. This tutorial account aims to provide the aerospace systems and reliability engineer with an elementary but rigorous introduction to these procedures.

INTRODUCTION

Need for Error Analysis

In computer-aided analysis of electrical circuits and similar systems, equations are supplied in the form

$$H(X_1, X_2, X_3\ \ldots) = \sum (-1)^{a_0} X_1^{a_1} X_2^{a_2} X_3^{a_3} \tag{1}$$

where a_i's are zero or unity, and X_i's are either unknown or known parameters, and the summation is over all possible combinations of the a_i exponents. A typical example would be

$$H(A,B,C,D) = A + BC + ABC - D \tag{2}$$

where A is an unknown quantity (such as voltage gain), and B, C, and D are known parameters (such as resistances, capacitances, and inductances). The aim is to take such an equation and, by treating it symbolically on a computer, to reach significant results without much numerical computation. One such result is an evaluation of the error which would occur if one of the known parameters became either zero or infinite. In the analysis of a system, in order to identify the component in which the error

is occurring, it is essential to have a computer-generated expression for the anticipated error. In addition to its use in failure diagnostics, knowledge of errors serves an additional purpose in deriving approximate mathematical models.

An Illustrative Example

One example should show how an equation such as Eq. (1) might arise. Consider a simple circuit in which resistances R_2 and R_3 are in series; the combination in parallel with R_1. The equivalent is

$$\overline{G} = \frac{1}{R_1} + \frac{1}{R_2 + R_3} = \frac{R_1 + R_2 + R_3}{R_1R_2 + R_1R_3} \tag{3}$$

or, in homogeneous form, which can be used by a computer

$$H(\overline{G}) = \overline{G}R_1R_2 + \overline{G}R_1R_3 - R_1 - R_2 - R_3 = 0 \ . \tag{4}$$

Equation (4) is now in the form of Eq. (1) with $\overline{G}$ as an unknown, and R_1, R_2, and R_3 all known parameters. The crucial questions then become:

1. What happens if, say, $R_2 = 0$ (a short circuit)?

2. What happens if $R_2 = \infty$ (an open circuit)?

These deviations would cause the greatest possible error in the unknown $\overline{G}$, and a method is needed to derive the error by computer-oriented calculation. This procedure is referred to as "tagging."

One-Variable Tagging

First, take a simplification of Eq. (1) where there is only one variable, P

$$H(P) = A + BP = 0 \tag{5}$$

To store such an equation in the computer, it is convenient to store the coefficient A with a tag of "0," indicating that P is not present, and to store the coefficient B with a tag of "1," indicating that P is present. The advantage of "tagging" is that significant results can be obtained from the computer by simply manipulating symbols rather than performing calculations.

Expansion in One Variable

Define the sum of the coefficients with tag "0" as $H(\overline{P})$, and define the sum of the coefficients with tag "1" as $H(P')$. Thus, the basic equation is written as:

$$H(P) = H(\overline{P}) + PH(P') \tag{6}$$

which corresponds exactly to Eq. (5) except for the constraint $H = 0$. This constraint is, however, always given and is the starting point for this type of computer-aided analysis. Unless otherwise stated, $H = 0$ will be assumed throughout this discussion.

Two simple relations can now be derived. Solving $H = 0$ for P,

$$P = \frac{-H(\overline{P})}{H(P')} \tag{7}$$

and differentiating Eq. (6) with respect to the variable P,

$$\frac{dH(P)}{dP} = H(P') \tag{8}$$

Two or More Variables

Expanding $H = 0$ to two variables, the basic equation becomes

$$H(P,Q) = H(\overline{P},\overline{Q}) + PH(P',\overline{Q}) + QH(\overline{P},Q') + PQH(P',Q') \tag{9}$$

which corresponds to the bilinear equation

$$H(P,Q) = A_0 + A_1P + A_2Q + A_3PQ \tag{10}$$

so that $H(\overline{P},\overline{Q})$ denotes terms "devoid" of both P and Q, $H(P',\overline{Q})$ denotes terms containing, but "deprived" of P, while "devoid" of Q, and so on.

The three-variable equation is therefore

$$H(P,Q,R) = H(\bar{P},\bar{Q},\bar{R}) + PH(P',\bar{Q},\bar{R}) + QH(\bar{P},Q',\bar{R}) + RH(\bar{P},\bar{Q},R') + PQH(P',Q',\bar{R}) + PRH(P',\bar{Q},R') + QRH(\bar{P},Q',R') + PQRH(P',Q',R') \ . \tag{11}$$

For example, if

$$H(P,Q,R) = S + STPQ - TQR + ZR,$$

then

$$H(\bar{P},\bar{Q},\bar{R}) = S \qquad H(\bar{P},Q',R') = -T$$

$$H(P',Q',\bar{R}) = ST \qquad H(\bar{P},\bar{Q},R') = Z$$

and the rest are zero.

THE SYMBOLIC ERROR FUNCTION

Motivation

Suppose $H(P) = 0$ with a single unknown P. Without explicitly solving, it is desirable to know, in some sense, what would be the error in P (or $1/P$) if one or more of the parameters were set to 0 or ∞.

Let $H(P,Q) = 0$, with P unknown and Q known. What happens to P if $Q = 0$? Setting $Q = 0$,

$$H(P,Q)\Big|_{Q=0} = H(\bar{Q}) = H(\bar{P},\bar{Q}) + PH(P',\bar{Q}) \tag{12}$$

and since $H = 0$,

$$P\Big|_{Q=0} = \frac{-H(\bar{P},\bar{Q})}{H(P',\bar{Q})} \triangleq P(\bar{Q}) \tag{13}$$

so that $P(\overline{Q})$ denotes P when Q = 0. Notice that Eq. (13) can be obtained from Eq. (7) by expanding and setting Q = 0.

Definition

In computer problems for which $H(P,Q) = 0$, it is more convenient to work with 1/P rather than P in order to facilitate further calculations. The symbolic error function is therefore defined as the error in 1/P:

$$E(\overline{P},\overline{Q}) \triangleq \frac{\frac{1}{P(\overline{Q})} - \frac{1}{P}}{\frac{1}{P}} . \tag{14}$$

It follows that

$$E(\overline{P},\overline{Q}) = \frac{P - P(\overline{Q})}{P(\overline{Q})} = \frac{P}{P(\overline{Q})} - 1 = \frac{H(\overline{P})}{H(P')} \cdot \frac{H(P',\overline{Q})}{H(\overline{P},\overline{Q})} - 1 . \tag{15}$$

Similarly, it is possible to define the error which occurs when $Q = \infty$. Dividing $H(P,Q) = 0$ through by Q, and letting $1/P = 0$

$$H(P,Q)\Big|_{Q=\infty} = QH(Q') = Q[H(\overline{P},Q') + PH(P',Q')] \tag{16}$$

and, since H = 0,

$$P\Big|_{Q=\infty} \triangleq P(Q') = \frac{-H(\overline{P},Q')}{H(P',Q')} . \tag{17}$$

This error function is therefore

$$E(\overline{P},Q') = \frac{\frac{1}{P(Q')} - \frac{1}{P}}{\frac{1}{P}} \tag{18}$$

so that

$$E(P,Q') = \frac{P - P(Q')}{P(Q')} = \frac{H(\overline{P})}{H(P')} \cdot \frac{H(P',Q')}{H(\overline{P},Q')} - 1 \, . \tag{19}$$

Single-Variable Reciprocal Equation

Divide $H(P) = H(\overline{P}) + PH(P') = 0$ by P on both sides, giving

$$\frac{1}{P} H(\overline{P}) + H(P') = 0 \, . \tag{20}$$

Let $R = 1/P$, transforming Eq. (20) into a linear equation in R.

$$H(P') + RH(\overline{P}) = 0 \tag{21}$$

By definition, therefore, $H(P') = H(\overline{R})$ because it is "devoid" of R, and $H(\overline{P}) = H(R')$ because it is "deprived" of R.

Definition of Reciprocal Error Function

Define the error function for $R = 1/P$, by

$$E(\overline{R},\overline{Q}) \triangleq \frac{\frac{1}{R(\overline{Q})} - \frac{1}{R}}{\frac{1}{R}} \, . \tag{22}$$

Thus,

$$E(\overline{R},\overline{Q}) = \frac{P(\overline{Q}) - P}{P} = \frac{H(P')}{H(\overline{P})} \cdot \frac{H(\overline{P},\overline{Q})}{H(P',\overline{Q})} - 1 \, . \tag{23}$$

Similarly

$$E(\overline{R},Q') \triangleq \frac{\frac{1}{R(Q')} - \frac{1}{R}}{\frac{1}{R}} \tag{24}$$

and

$$E(\overline{R},Q') = \frac{H(P')}{H(\overline{P})} \cdot \frac{H(\overline{P},Q')}{H(P',Q')} - 1 \tag{25}$$

$E(\overline{R},\overline{Q})$ and $E(\overline{R},Q')$ are called reciprocal error functions to $E(\overline{P},\overline{Q})$ and $E(\overline{P},Q')$.

Two definitions now complete the general definition of the error function

$$E(P',\overline{Q}) \triangleq P \cdot E(\overline{R},\overline{Q}) = P(\overline{Q}) - P \tag{26}$$

$$E(P',Q') \triangleq P \cdot E(\overline{R},Q') = P(Q') - P \tag{27}$$

Now, Eqs. (14), (18), (26), and (27) provide a general definition of $E(X,Y)$, where X is either $\overline{P}$ or P', and Y is either $\overline{Q}$ or Q'. A bar in the second position denotes an error caused by a parameter going to zero, while an apostrophe denotes an error caused by a parameter going to infinity. A bar in the first position denotes error in the reciprocal of the variable, while an apostrophe denotes the error in the variable itself.

SKETCH OF THE CALCULUS OF SYMBOLIC ERROR FUNCTIONS

A Useful Theorem

All of the above definitions are special cases of a general relation

$$E(X,Y) = \frac{-H(Y)}{H(X,Y)}$$

To prove this, take one case at a time, as follows

(1) $$E(\overline{P},\overline{Q}) = \frac{P - P(\overline{Q})}{P(\overline{Q})}$$

$$= \frac{PH(P',\overline{Q}) + H(\overline{P},\overline{Q})}{-H(\overline{P},\overline{Q})} = \frac{-H(\overline{Q})}{H(\overline{P},\overline{Q})}$$

by substituting $P(\overline{Q}) = \dfrac{-H(\overline{P},\overline{Q})}{H(P',\overline{Q})}$

$$(2) \quad E(P',\overline{Q}) = P(\overline{Q}) - P$$

$$= \frac{H(\overline{P},\overline{Q}) + PH(P'\overline{Q})}{-H(P',\overline{Q})}$$

$$= \frac{-H(\overline{Q})}{H(P',\overline{Q})}$$

The proofs for $E(\overline{P},Q')$ and $E(P', Q')$ are obtained by replacing $\overline{Q}$ by Q' in the above.

The significance of this result is that it supplies a simpler expression which holds for all cases of the single-parameter error function.

Multi-Variable Errors

Consider $H(P,Q,S) = 0$. Let

$$P\Big|_{\substack{Q=0\\S=0}} \triangleq P(\overline{Q},\overline{S}) = \frac{-H(\overline{P},\overline{Q},\overline{S})}{H(P',\overline{Q},\overline{S})} \tag{28}$$

where the last equality is obtained by substituting $Q = S = 0$ in $H(P,Q,S) = 0$. Similarly,

$$P\Big|_{\substack{Q=0\\S=\infty}} \triangleq P(\overline{Q},S') = \frac{-H(\overline{P},\overline{Q},S')}{H(P',\overline{Q},S')} \tag{29}$$

where the last equality is obtained by dividing $H(P,Q,S) = 0$ by S on both sides and substituting $Q = 1/S = 0$.

This type of definition can be extended to failures in as many parameters as desired.

For example, if $H(A,B,C,D,E,F) = 0$,

$$A(\overline{C},\overline{D},F') = \frac{-H(\overline{A},\overline{C},D',F')}{H(A',\overline{C},D',F')} \tag{30}$$

The error functions for $H(P,Q_1,Q_2,\ldots Q_n) = 0$ are defined by

$$E[\overline{P},(X_1,X_2,\ldots,X_n)] = \frac{\dfrac{1}{P(X_1,X_2,\ldots X_n)} - \dfrac{1}{\overline{P}}}{\dfrac{1}{\overline{P}}} \tag{31}$$

$$E[P',(X_1,X_2,\ldots,X_n)] = P(X_1,X_2,\ldots,X_n) - P \tag{32}$$

where the X_l are of the form $\overline{Q}_l$ or Q_l'.

Consider $H(P,Q_1,\ldots,Q_n) = 0$. The general relation satisfied by all error functions is

$$E[X,(Y_1,Y_2,\ldots,Y_n)] = \frac{-H(Y_1,Y_2,\ldots,Y_n)}{H(X,Y_1,Y_2,\ldots,Y_n)} \tag{33}$$

where

$$X = \overline{P} \text{ or } X = P' \quad \text{and} \quad Y_l = \overline{Q}_l \text{ or } Y_l = Q_l' .$$

The proof is a simple extension of the one-variable case, using the definitions of $E[X(Y_1,\ldots,Y_n)]$ and $X(Y_1,\ldots,Y_n)$.

Higher-Order Errors

It is useful to define the "error of an error":

$$E[E(X,Y),Z] \triangleq \frac{E[X(Y,Z)] - E(X,Y)}{E(X,Y)} \tag{34}$$

For example:

$$E[E(\overline{P},\overline{Q}),S] = \frac{E[\overline{P},(\overline{Q},S')] - E(\overline{P},\overline{Q})}{E(\overline{P},\overline{Q})}$$

The following identity:

$$E[E(X,Y),Z] = \frac{E[Z,(X,Y)]}{E(Z,Y)} - 1 \qquad (35)$$

can be proved by letting $H(P,Q,S) = 0$, with $X = \overline{P}$ or P', $Y = \overline{Q}$ or Q', $Z = \overline{S}$ or S', and using the identities

$$E(X,Y) = \frac{-H(Y)}{H(X,Y)} \qquad E[Z,(X,Y)] = \frac{-H(X,Y)}{H(X,Y,Z)}$$

and the definition of $E[E(X,Y),Z]$.

Electronics Research Center
National Aeronautics and Space Administration
Cambridge, Massachusetts, December 1968
125-25-04-63

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